International Journal of Engineering Technology and Applied Science

(ISSN: 2395 3853), Vol. 3 Issue 5 May 2017

Modelling and Finite Element Analysis of Dynamics of Open Crack Rotor

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Abstract- The critical and rotational speed analysis at natural frequency of rotating systems is pronounced as a key perform altogether the fields of engineering. The behavior of the rotor systems are primarily ensuing from the excitations from its rotating parts. There are many numerical ways present to research the rotor-bearing systems. Finite part methodology may be a key tool for dynamic analysis of rotor bearing system. This study describes a multi disk, variable cross section rotor-bearing system with cross crack on axis isosceles parts supported on bearings in an exceedingly mounted frame. The shaft within the rotor-bearing system is assumed to adapt Euler Bernoulli beam theory. The equation of motion of the rotor-bearing system springs by Lagrangian approach alongside finite part methodology. Finite part model is employed for vibration analysis by together with rotary inertia and rotating mechanism moments with consistent matrix approach. The rotor bearing system consists of 2 bearings and 2 rigid disks. One disk is overhung and also the alternative one is placed between the bearings. Internal damping of the shaft and linear stiffness parameter of the bearings are taken under consideration to get the response of the rotor-bearing system. The rotor has variable crosswise throughout the configuration. The disks are sculptured as rigid and have mass unbalance forces. The crucial speed, unbalance response and natural frequency are analyzed for the standard rotor-bearing system with cross crack. Analysis includes the impact of crack depths, crack location and spin speed. The results are compared with the results obtained from finite part analysis. The bearing configurations are un-damped isotropic and orthotropic. The natural whirl speeds are analyzed for the synchronous rotor for each the un-cracked and cracked rotor bearing system exploitation Joseph Campbell diagrams.

KEYWORDS: Cambell Diagram, FEM, FEA, Open Crack, Crack Depth.

I INTRODUCTION

From ISO definition, rotor can be defined as a body which is suspended through a set of cylindrical rest or bearings that grants the system to rotate freely about an axis secured in space. In the basic level of rotor dynamics, it is related with one or more mechanical structures (rotors) supported by bearings that rotate around a unique axis. The non-spinning structure is called a stator. When the spin speed increases the amplitude of vibration increases and is maximum at a speed called critical speed. This amplitude is often elevated by unbalance forces from disk of the spinning system. When the system reaches excessive amplitude of vibration at the critical speed, catastrophic failure occurs. Normally turbo machineries frequently develop instabilities which are mainly due to the internal configuration, and should be rectified. Often rotating structures originates vibrations depending upon the complexity of the mechanism involved in the process. Even a small misalignment in the machine can increase or excite the vibration signatures. System vibration behaviour due to imbalance is the main aspects of rotating machinery, and it must be measured in detail and reviewed while designing. Every object including rotating structures shows natural frequency depending on the complexity of the structure. The critical speed of these rotating structures arises when the rotational speed meets with its natural frequency. The first critical speed can be encountered at the lowest speed. However as the speed increases further critical speeds can also be spotted. It is very essential to reduce the rotational unbalance and excessive external forces to minimize the overall forces which actuate resonance. The major concern of designing a rotating machine is, avoiding the vibration in resonance which creates a destructive energy. Situations involving rotation of shaft near critical speed must be avoided. When these aspects are ignored it might results in wear and tear of the equipment, failure of the machinery, human injury and sometime cost of lives.

1.3 Basic Principle

To model the actual dynamics of the machine theoretically is a cumbersome task. Based on the simplified models, the calculations are made to simulate various structural components. The dynamic system of equations have interesting feature, in which the off-diagonal elements are stiffness, damping, and mass. These three elements can be called as, cross-coupled stiffness, cross-coupled damping, and cross-coupled mass. Although there is a positive cross-coupled stiffness, a deflection will originate a reaction force opposite to the direction of deflection, and also the reaction force can be in the direction of positive whirl. When these forces are large compared to the direct damping and stiffness, the rotor will be unstable. If a rotor is unstable it typically needs a prompt shutdown of the machine to avoid breakdown. In figure 1 shows the general principle of a rotor mounted on two bearing with a single plane motion in Y-Z direction.

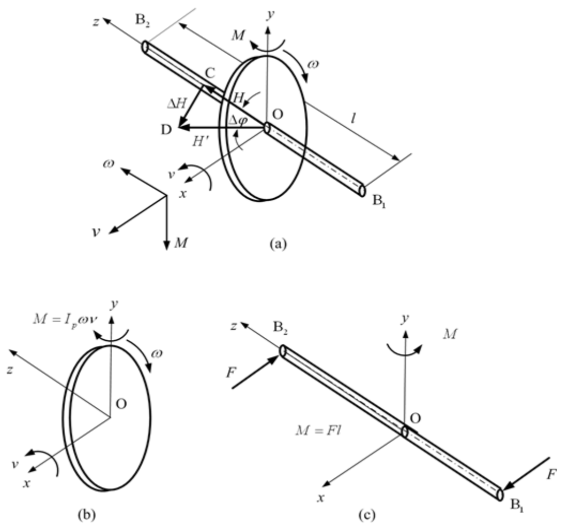

Figure 1 General Principle of a Rotor Mounted on Two Bearing with a Single Plane Motion in Y-Z Direction.

II LITERATURE SURVEY

In 2016 Elsevier Huichun peng et al. [1] proposed that the damping effects with the distinction of stationary damping and the anisotropic rotating damping on the dynamic stability of the rotating rotor with an open crack on the surface of the shaft is studied. The motion equations of the cracked rotor system are formed by Lagranges principal. Different from previous studies, the anisotropic system with the multi periodical varied coefficients is simplified by the moving frame method such that the stability analysis based on the root locus method can be applied. The corresponding Campbell diagram, decay rate plot and roots locus plot are derived to prove the destabilizing influence of both the rotational damping and the varied anisotropy ratio of the rotating damping. The effects of anisotropy of stiffness on the decisions of the critical range are also presented. The result with theoretical precision would not only generally provide practical applicability to crack detection and instability control of the heavy loading turbo-machinery system, but also give the suggestion that, the increased proportion and the aggravated anisotropy of the rotational damping due to the crack of the fatigue rotor should been taken into consideration on the modeling of cracked rotor system.

In 2016 Engineering Solid Mechanics Growing Science R. Tamrakar et al. [2] proposed that the, vibrational response of a cracked rotor in static and rotating condition through Campbell diagram. An open crack in the rotor changes its stiffness. The effect of which is seen on the natural frequency of the system. The natural frequency of the cracked rotor increases in comparison to un-cracked rotor. Experimental and simulation work is performed in the static condition to study the natural frequency of the rotor. Campbell diagram is generated through Simulation in ANSYS to study the critical speed variation at first (I) and second (II) Engine order (EO) line for cracked and un-cracked rotor.

In 2016 MOVIC & RASD Zhiwei Huang et al. [3] presented that the Rub-impact and fatigue crack are two important rotor faults. Based on the crack theory, an improved switching crack model is presented. Dynamic characteristics of a rotor-bearing system with imbalance, rub-impact and transverse crack are attempted. Various nonlinear dynamic phenomena are analyzed using numerical method. The results reveal that unstable form of the rotor system with coupling faults is extremely complex as the rotating speed increases and there are some low frequencies with large amplitude. The influence of crack depth and angle on the dynamic behavior of a cracked rotor is important insomuch as the quasi-periodical and chaotic motions in system response will occur along with different parameters. It is indicated that this study can contribute to a further understanding of the non-linear dynamics of such a rotor system with coupling faults of crack and rub-impact.

In 2016 Science Direct Anuj Kumar Jain et al. [4] proposed an article. In this proposed article, the dynamic behavior and diagnostic of cracked rotor have been gained momentum. In literature, several studies are available for cracked rotor systems, however very few authors have addressed the issue of multi-cracked rotor system. This paper deals with the nonlinear dynamic behavior of multi cracked rotor system, which is analyzed experimentally and analytically with the considerations of the effects of the crack depth, crack location and the shaft’s rotational speed. A new extension of Lagrangian method is used for analyzing the dynamic behavior of a multi-cracked rotor system through Umbra Lagrangian formalism. The effects of crack depth on the shaft’s stiffness and natural frequencies are analyzed experimentally. Natural frequencies have been obtained through vibration analyzer using impact hammer test under static conditions. This analysis also includes the dynamic response of rotor with breathing crack by using data acquisition system called OROS. It has been noticed that the stress concentration on the first crack has increased due to the presence of the second crack. Another interesting phenomenon is the influence of one crack over the other crack for mode shapes and for threshold speed limits. All such analysis has been carried out on experimental test rig consist of a symmetrical system, which has mild steel shaft between a pair of identical self-aligned double groove high speed bearings. The experimental results can be further validated with the analytical results.

In 2013 M. Serier et al [5] proposed that the design of experiment method is used to investigate and explain the effects of the rotor parameters on crack breathing and propagation in the shaft. Three factors are considered which have an influence on the behavior and the propagation of the crack: the rotational speed, the length of the rotor and the diameter of the shaft. The elaborated mathematical model allows determining the effects and interaction of speed, diameter and length on crack breathing mechanism. The model also determines the optimal values of the parameters to achieve high performance.

III FINITE ELEMENT ANALYSIS

FEA stands for Finite Element Analysis and as the name suggest the methodology involves the analysis of finite elements. The whole model is divided into number of finite elements and then all the forces and boundary conditions are applied on these finite elements, and then the results of all these finite elements are combined together to give the output of whole model. For example if a line is representing a beam and we have to analyze that beam as a cantilever then FEA will divide this line representing a beam into number of small segments known as element. Then the effect of boundary condition and forces is studied on each segment and the resultant output is the summation of each segment. FEA analysis can help engineers to analyze complicated models. With the development of computer systems FEA has increased to gain importance, since it saves time and money both. Meshing of structure is done to analyse the model, which involves the division of structure into small finite elements. Figure 2 represents the block diagram of FEA analysis.

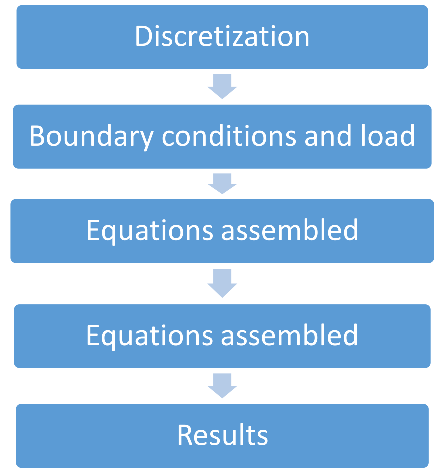

Figure 2 Block diagram of FEA process

Simple polynomial shape functions and nodal displacements are used to determine the overall displacement. Unknown nodal displacements are used to determine strains and stresses developed. From this, the equations of equilibrium are assembled in the matrix form which can be easily be programmed and solved through a computer program. After applying the boundary conditions and loads, nodal displacements are found by solving the matrix stiffness equation. After the nodal displacements are known, element stresses and strains can be calculated.

5 Boundary Condition

The rotor is applied to the fixed to fixed support at the hub to restrict its motion along any axis.

- **Load**
  - (i) Centrifugal force is applied along Y-axis (Clockwise) to the rotor.
  - (ii) Earth gravity is applied about Y-axis in negative direction.
- **Analysis and Results**
  - (i) Element type -3D solid element (Tetrahedron mesh)
  - (ii) Number of element- 5057
  - (iii) Number of nodes- 2497
- **Material used – Mild Steel and Cast Iron**
  - (i) Density : 7,800 kg/m^3 (0.28 lb/cu in)
  - (ii) Young's Modulus: 200 G-Pa
  - (iii) Poisson's ratio : 0.26
  - (iv) Shear Modulus : 75 G-Pa
- **Loading**
  - (i) Centrifugal load is applied to the compressor along Y-axis (Clockwise) having angular velocity ω = 3663 rad/s.
  - (ii) Earth gravity is applied about Y-axis, g= -9806.6 mm/s^2.

Transverse crack element modelling

For an open crack case, the stiffness matrix of the cracked element in a generalized form similar to that of the asymmetric rod can be written as;

$$k_{ce}^{j}(t)=\frac{E}{l^3}\begin{bmatrix} 12I\bar{y}(t) & 0 & 0 & -2I\bar{y}(t) & -12I\bar{y}(t) & 0 & 0 & -6I\bar{y}(t) \\ 0 & 12I\bar{z}(t) & 6I\bar{z}(t) & 0 & 0 & -12I\bar{z}(t) & 6I\bar{z}(t) & 0 \\ 0 & 6I\bar{z}(t) & 4I\bar{z}(t) & 0 & 0 & -6I\bar{z}(t) & 2I\bar{z}(t) & 0 \\ -6I\bar{y}(t) & 0 & 0 & 4I\bar{y}(t) & 6I\bar{y}(t) & 0 & 0 & 2I\bar{y}(t) \\ -12Iy(t) & 0 & 0 & 6Iy(t) & 12Iy(t) & 0 & 0 & 6I\bar{y}(t) \\ 0 & -12I\bar{z}(t) & -6I\bar{z}(t) & 0 & 0 & 12I\bar{z}(t) & -6I\bar{z}(t) & 0 \\ 0 & 6I\bar{z}(t) & 2I\bar{z}(t) & 0 & 0 & 6I\bar{z}(t) & 4I\bar{z}(t) & 0 \\ -6I\bar{y}(t) & 0 & 0 & 2I\bar{y}(t) & 6I\bar{y}(t) & 0 & 0 & 4I\bar{y}(t) \end{bmatrix}$$

From equation, l represents the element length, E is the elastic modulus. The expressions for the time-varying quantities $I\bar{y}(t)$ and $I\bar{z}(t)$ are given in the following consequent sections.

Equation of motion of the system with transverse open crack

The global equation of motion for the rotor-bearing system with the transverse open crack can be written in fixed frame coordinates by neglecting the unbalance force as;

$$[M^S]\{\ddot{q}^S(t)\}-\Omega[G^S]\{\dot{q}^S(t)\}+\left([K^S]+\left[\tilde{K}(t)\right]\right)\{q^S(t)\}=0$$

Where, $[M^S]$ = global mass matrix.

$[G^S]$ = global gyroscopic matrix.

$[K^S]$ = global stiffness of the un-cracked rotor bearing system equal to k_{01}^{j}.

$\left[\tilde{K}(t)\right]=[\tilde{K}_0]cos\left(2(\Omega t+\phi)\right)$

$[\tilde{K}_0]$ = global stiffness matrix of the cracked element, equal to k_{01}^{j}.

$\{q^S(t)\}=\{q_1^e \ldots\ldots\ldots q_i^e \ldots\ldots\ldots q_{N+1}^e\}^T$ Is the global displacement vector.

The matrices $[M^S], [G^S], [K^S]$ and $\left[\tilde{K}(t)\right]$ are having the dimension of 4(N+1) × 4(N+1). The equation of motion of the system is a second order differential equation with frequency of 2 Ω for the open crack. This system is periodically time varying.

IV RESULTS AND DISCUSSION

Critical Speed and Frequency along the Shaft with Different Diameter and Materials

A Modal - analysis was carried out to analyse critical speed of shaft with Notch and shaft without Notch with two different material by using Campbell diagram and relation between natural frequency and spin speed and at two types of materials of Mild steel and Cast iron to determine the frequency distribution along the Shaft. Frequency distribution contours in case of different diameter for the two different profiles are shown in Figure, and the effect of different shaft profiles on the frequency and modes distribution for various different diameter and materials are represented in the Figure 3.

Analysis of Notch Shaft and without notch shaft with different materials

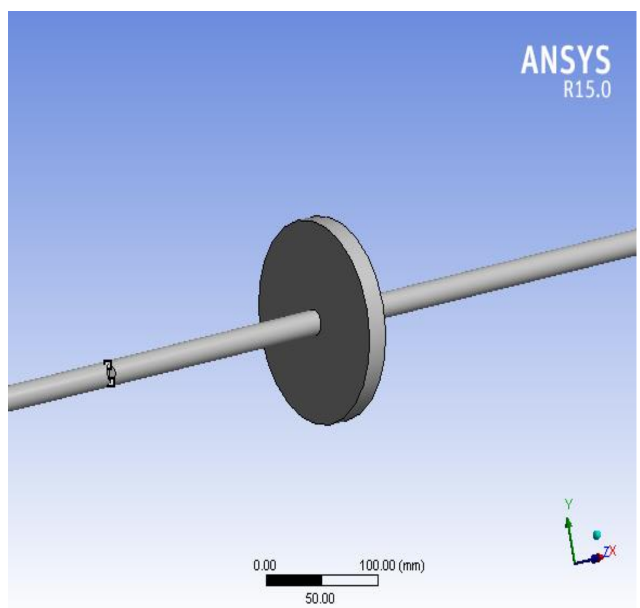

Figure 3 CAD model of Notch Shaft

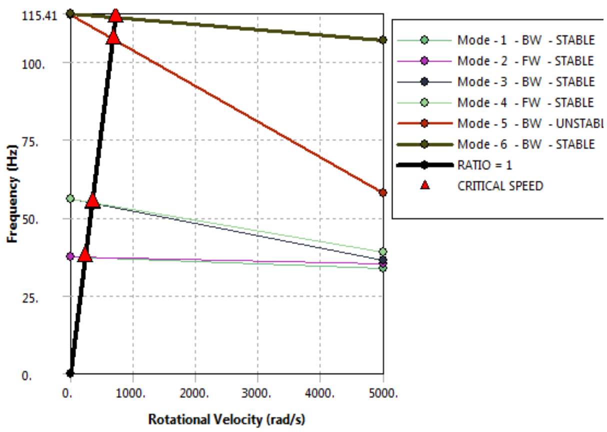

Figure 4 Result of Campbell diagram of frequency and rotational velocity distributions along the Mild steel with Notch shaft

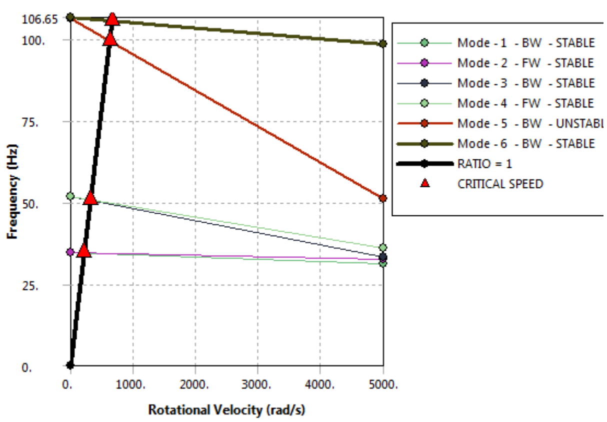

Figure 5 Result of Campbell diagram of frequency and rotational velocity distributions along the Cast iron with Notch shaft

Table No. 1 Critical Speed of Notch Shaft

Critical Speed			
Modes	Shaft Stability	Mild Steel	Cast Iron
1	Stable	234.70	216.86
2	Stable	235.32	217.49
3	Stable	343.77	317.93
4	Stable	345.20	319.26
5	Unstable	673.59	623.79

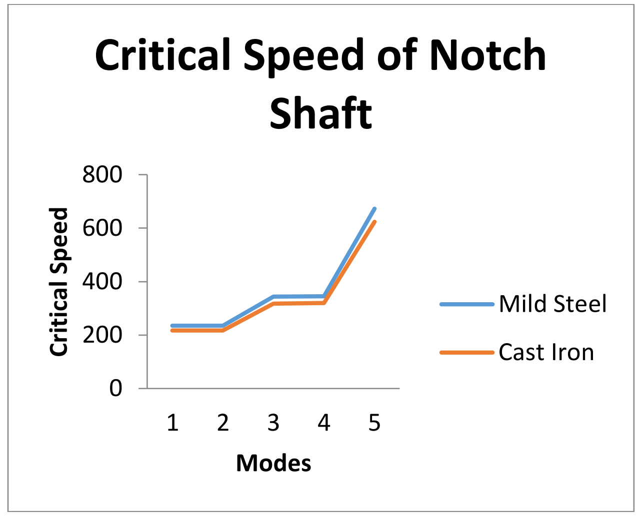

Figure 6 Graph shows modes and Critical Speed of Notch Shaft

The above table 1 & graph in figure 6 shows the critical speed values and comparison of three materials with respect to modes on constant diameter shaft, the result shows that grey cast iron seems to be less critical speed due to its high stiffness and lesser deformation at different whirling modes.

Analysis of Shaft without notch with Different Materials

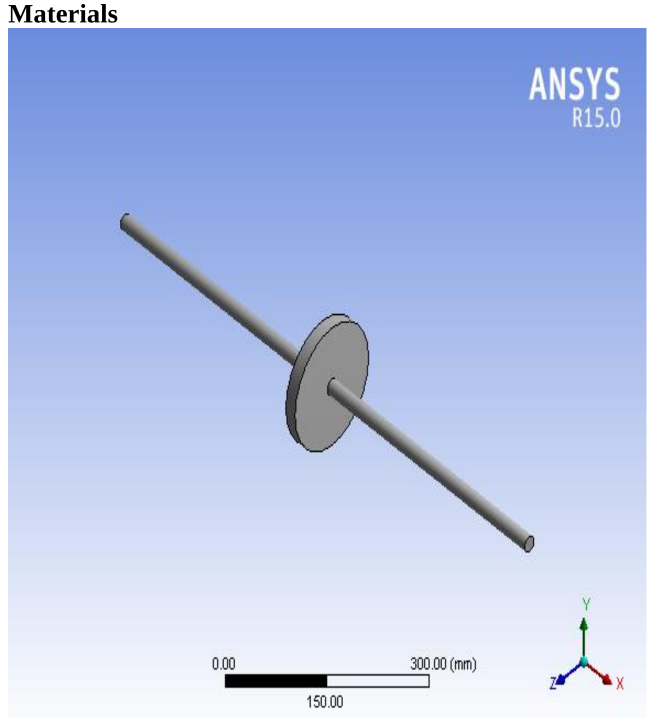

Figure 7 CAD model of Shaft without notch.

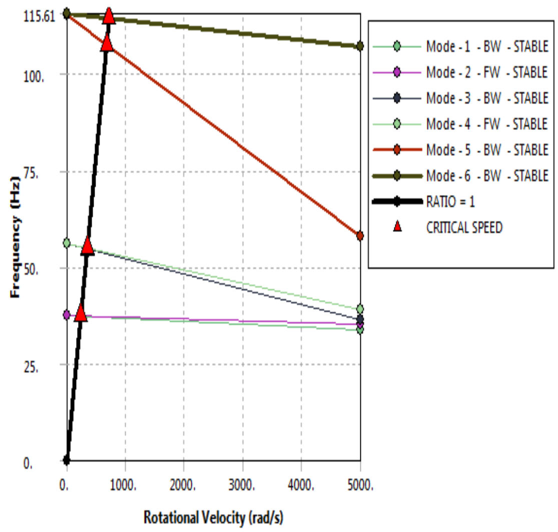

Figure 8 Result of Campbell diagram of frequency and rotational velocity distributions along the Mild steel shaft without notch

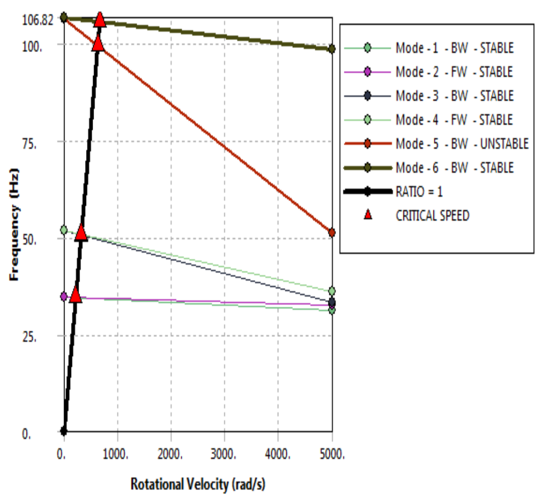

Figure 9 Result of Campbell diagram of frequency and rotational velocity distributions along the cast iron Shaft without notch

Table No. 2 Critical Speed of Shaft without Notch

Critical Speed			
Modes	Shaft Stability	Mild Steel	Cast Iron
1	Stable	234.93	217.09
2	Stable	235.66	217.78
3	Stable	344.10	318.23
4	Stable	345.72	319.70
5	Unstable	674.68	624.81

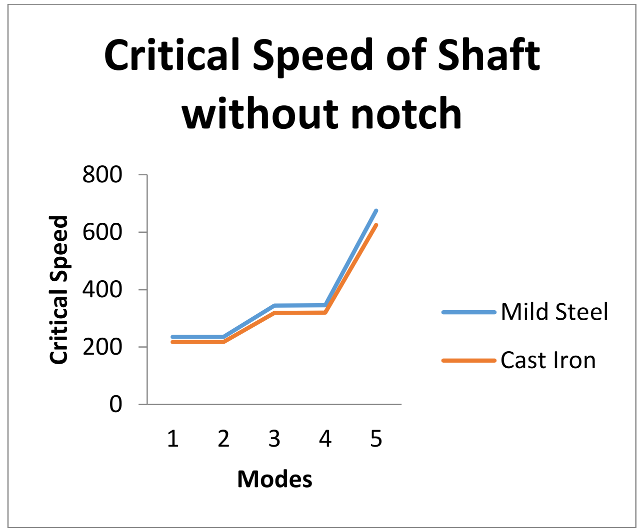

Figure 10 Graph shows modes and Critical Speed of Shaft without Notch

The above table 2 & graph in figure 10 shows the critical speed values and comparison of four materials with respect to modes on stepped shaft 70 mm-100 mm diameter, grey cast iron seems to be less critical speed due to its high stiffness and lesser deformation at different whirling modes.

Table No. 3 Natural Frequency with their Different Modes with Notch shaft

Natural Frequency		
Mode	Mild steel	Cast iron
1	37.525	34.667
2	37.565	34.706
3	56.076	51.769
4	56.129	51.82
5	114.88	106.15
6	115.41	106.65

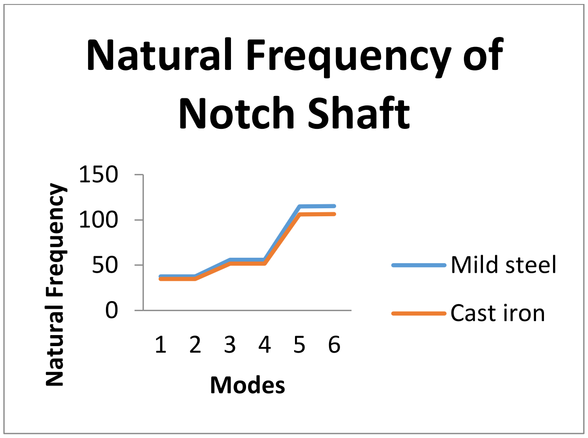

Figure 11 Graph shows modes and frequency of notch shaft

The above table 3 & graph 11 shows the natural frequency values of two materials with respect to modes on notch shaft, both materials were compared with respect to their modes.

In case of stepped shaft the diameter changes hence frequency increases the deformations of shaft shown in contour plots are w.r.t. frequency at constant angular velocity of 3663 rad/s these deformation changes as per modes of frequency that at particular section the value of frequency is high and low this frequency is damped natural frequency of shaft at synchronous speed which occurs due to mass imbalance also the modal damping ratio was considered it is a ratio of damping constant to the damped natural frequency from below contour plots the values obtained are compared in graphs and was predicted that in every material damped natural frequency increases, Hence modal damping ratio decreases from this effect logarithmic decrement increases that means decrease in amplitudes.

Table No. 4 Natural Frequency with their different modes of shaft without notch

	Natural Frequency	
Mode	**Mild Steel**	**Cast iron**
1	37.562	34.703
2	37.620	34.753
3	56.131	51.820
4	56.217	51.893
5	115.09	106.34
6	115.61	106.82

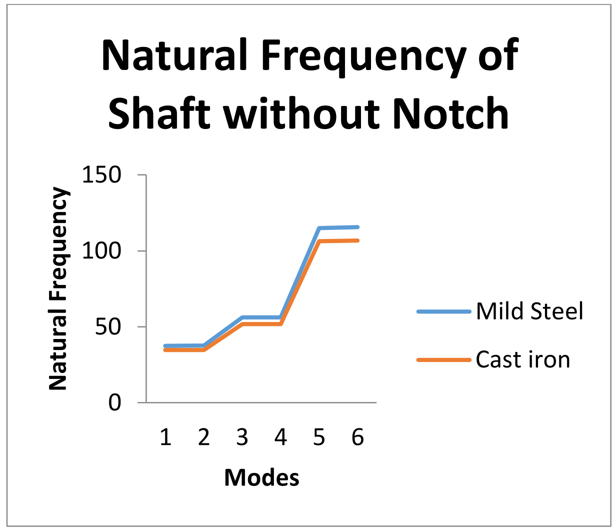

Figure 12 Graph shows modes and frequency of shaft without Notch

The above table 4 & graph in figure 12 shows the natural frequency values of two materials with respect to modes on shaft without Notch, both materials were compared with respect to their modes.

Discussion

It is evident that there is an increase in the frequency along the shaft for all the two profiles with various different diameters. It is found that the mild steel has maximum frequency in the case of profile and minimum for cast iron profile. This is because different diameter value will lead to more frequency being transferred to the material and less vibration stored in the shaft, hence resulting in low vibration and higher frequency base. This reduced vibration will induce smaller resonance condition and the consequent increase in the frequency of different material with shaft of different profiles.

V CONCLUSION

The current analysis has presented a study of natural frequency characteristics of a shaft of different profiles. Modal analysis was carried out on Mild steel and Cast iron system. The effect of shaft with notch and shaft without notch profiles of the Shaft on the natural frequency and modes of different materials and critical speed effects were analysed on different profile and materials of shaft and distribution along the shaft was studied. From the analysis of the results, following conclusions can be drawn.

Influence of different shaft profiles

- The natural frequency along the shaft profile is found to be maximum of the mild steel material profile with shaft without notch and shaft with notch are varies along the length up to the shaft for all the two profiles. The critical speed distribution along the shaft is maximum for mild steel and minimum for cast iron of a shaft without notch profiles.

- The magnitude of frequency is maximum in the case of mild steel material profile of shaft with notch and shaft without notch. The nature of the natural frequency is maximum near its end in 3rd and 5th, 6th mode.
- The nature of the critical speed is maximum near its masses and between the end of the shaft where masses are placed of shaft and changes with respect to shaft material with different profile towards the end and between masses of the shaft for the same RPM and different modes of natural frequency.

In a comparison with the Mild steel and Cast iron material resulted in higher frequency characteristics close to the end of the shaft for a different shaft diameter profile. The critical speeds are maximum for mild steel at high frequency and minimum for cast iron at less frequency on same RPM.

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