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COMMONE .A. LIKE PROPERTY ON FIXED POINT THE OREM WITH INTEGRAL TYPE IN EQUALITY IN FUZZY2-METRICSPACE

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ABSTRACT:

In this paper we use common E.A. like property with integral type inequality in fuzzy2 metric space and the condition for two pairs of weakly compatible maps under E.A. like property with integral type in equality have unique common fixed point.

Keywords: Fuzzy2-metricspaces, common fixed point, weakly compatible maps, common E.A. like property.

MATHEMATICS SUBJECT CLASSIFICATION:

47H10,54H25

INTRODUCTION:

Fuzzy set has been defined by Zadeh[13]in1965.In1975,Kramosil and Michalek [7] introduced the concept of a fuzzy metric space. Many authors extended their views as some George andVeeramani[6].Grabiec(1988),Subramanyan(1995),Vasuki(1999),PantandJha(2004)obtained some analogous results proved by Bal subramania metal. A amriand EL.Moutawakil [1] generalized the concepts of non-compatibility by defining the notion of (E.A.)property.

Gahlerin vestigated 2-Metricspaces in a series of his papers [3], [4],[5].

It is to be remarked that Sharma and Iseki [7] studied for the first time contraction type mappingin2metricspace.ChoS.H.[2] proved a common fixed point the orem for four mappings in fuzzy metric space and SharmaS.[10]proved a common fixed point the orem for three mapping in fuzzy2-metric space.In this paper we prove common fixed point the ore musing common E.A.like property with integral type in equality in fuzzy2-metric space.

2. PRELIMINARIES

Definition 2.1: Atrianglenorm$*$is a binary operation on the unit interval [0,1]such that for all a,,,$\in$[0,1].The following conditions are satisfied

(i) $a*1=a$,

(ii) $a*b=b*a$

(iii) $a*b\leq c*d$whenever$a\leq c and b\leq d$

(iv) $a*(b*c)=(a*b)*c$

Definition 2.2[12]:A binary operation$*$:[0,1]×[0,1]×[0,1]→[0,1] is a continuous$t$-norm if([0,1]$*$)is an abelian topological monoid with unit1such that$a_1*b_1*c_1\leq a_2*b_2*c_2$whenever$a_1\leq a_2,b_1\leq b_2,c_1\leq c_2$forall$a_1,a_2,b_1,b_2andc_1,c_2\in[0,1]$.

Example of at-norm are $a*b=ab$ and $a*b$= min{a, b}

Definition 2.3[12]:A 3-tuple(X,M,$*$) is saidto befuzzy2-metric spaceifX is an arbitrary set, $*$is a continuoust-norm and Misfuzzy set sin$X^3\times[0,\infty)$satisfying the following conditions,for allx,y, $z,u\in$X andt_1,t_2,t_3> 0.

(i) M(x,y,z,0)= 0,

(ii) M (x,,,)=1forall$t$> 0ifand only if atleastt woor three points are equal,

(iii)M(x,y,z,t)=M(x,z,y,t)=M(y,z,x,t)

(iv)M($x,y,z,t_1+t_2+t_3$)≥M(x,y,u,t_1)$*$M(x,u,z,t_2)$*$M(u,y,z,t_3)(This correspond to tetrah edron in equality in 2-metric space)

The function value M(x,y,z,t) may be interprete dasthe probability that the area of triangle is less than t.

(v)M (x,y,z, .) :[0, ∞)→[0,1]is left continuous.

Definition 2.4: Let $f$and$g$ beself mapping from a fuzzy2-metric space (X,M,$*$) into itself. A Pair of map{f,g}said to be compatible if $\lim_{n\to\infty}$ M (fgx_n,x_n,a,t)=1,whenever{x_n}is a sequence in X such that $\lim_{n\to\infty} fx_n=\lim_{n\to\infty} gx_n=u$ for some u in X and for all t>0.

Definition 2.5: A pair of self mapping {f,g} of a fuzzy 2-metric space (X, M, $*$) is said to be weakly compatible if they commute at the coincidence points, i.e. fu= $gu$for some u $\in$Xthen$fgu=gfu$.

It is to see that two compatible maps are weakly compatible but converse is not true.

Definition 2.6: Let $f$and $g$be two self-maps of a fuzzy metric space (X, M,∗). Then they are said to satisfy the E. A. property, if there exists sequence$\{x_n\}$ in X such that

$\lim_{n\to\infty} f x_n = \lim_{n\to\infty} g x_n = t$ for some $t \in X$

Now inSimilarmodewestateE.A.propertyinfuzzy2-metricspaces as follows

Definition 2.7: A pair of self mapping $\{f,\}$ofafuzzy2-metricspace(X,M,∗)issaidtobe E.A .property, if there exists sequence$\{x_n\}$ in X such that

$\lim_{n\to\infty} M(f x_n, g x_n, a, t) = 1$ for some $t \in X$.

Definition 2.8 [11]: LetA,B,S, T: X→ X where X is afuzzy2-metricspace, then the pair{A,S} and {B,T} said to satisfy common E.A. like property if there exist two sequences$\{x_n\}$and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} A x_n = \lim_{n\to\infty} S x_n = \lim_{n\to\infty} B y_n = \lim_{n\to\infty} T y_n = z$$

Where$z \in S(X)\cap T(X)$ or$z \in A(X)\cap B(X)$

Lemma 2.9: Let(X,M,∗)be a fuzzy2-metricspace.If the reexist k∈(0,1)such that for all $M(x,,,kt) \geq M(x,y,z,t)$forall$x,y,z \in X$with$z \neq x, z \neq y$and $t>0$,then$x=y$.

Definition 2.10: LetΦbethesetofallrealcontinuousfunctions$F:[0,1]^2 \to [0,1]$non-decreasing in each coordinate variable and such that$F(t,1) \geq t, F(1,t) \geq t$,for all$t \in [0,1]$.

3. MAIN RESULT

The orem3.1:LetP,Q,R and S be self–maps of a fuzzy2- metric spaces(X,M,∗)satisfying the following condition:

(1) For any$x$, y, $z \in X$, and for all t > 0 there exists a number k ∈(0, 1) such that

$$\int_0^{M(Rx,Sy,z,kt)} \varphi(t)\,dt \geq \int^{\min\left\{ M(Px,Sy,z,t), \frac{1}{3}\left(M(Rx,Px,z,t)+M(Rx,Qy,z,t) + [M(Rx,Qy,z,t)*M(Qy,Px,z,t)] \right) \right\}} \varphi(t)\,dt$$

(2) Pairs(R, P) and (S, Q)satisfy common E. A. like property

(3) Pairs(R, P) and (S, Q)are weakly compatible.

Then P, Q,R and Shave a unique common fixed point in X.

Proof: Since (R, P) and (S, Q) satisfy common E. A. like property therefore there exist two sequences $\{x_n\}$ and$\{y_n\}$ in X such that

$\lim_{n\to\infty} Rx_n = \lim_{n\to\infty} Px_n = \lim_{n\to\infty} Sy_n = \lim_{n\to\infty} Qy_n = z_1$

Where$z_1 \in$ P(X)∩Q(X) or$z_1 \in$ R(X)∩S(X).

Suppose$z_1 \in$ P(X)∩ Q(X),now we have $\lim_{n\to\infty} Rx_n = z_1 \in$ P(X)then$z_1 = Su$for some $u \in$X

Now we claim that $Ru = Pu$,from(1),we have

$$\int_0^{M(Ru,Sy_n,z,kt)} \varphi(t)dt \geq \int_0^{\min\{\frac{1}{3}\left(\begin{matrix} M(Pu,Sy_n,z,t), \\ M(Ru,Pu,z,t)+M(Ru,Qy_n,z,t) \\ +[M(Ru,Qy_n,z,t)*M(Qy_n,Pu,z,t)]\end{matrix}\right)\}} \varphi(t)dt$$

Taking limit $n\to\infty$, we have

$$\int_0^{(Ru,_1,z,kt)} \varphi(t)dt \geq \int_0^{\min\{\frac{1}{3}\left(\begin{matrix} (z_{1},_1,z,t), \\ M(Ru,z_1,z,t)+M(Ru,z_1,z,t) \\ +[MRu,z_1,z,t*M(z_1,z_1,z,t)]\end{matrix}\right)\}} \varphi(t)dt$$

$$\int_0^{(Ru,_1,z,kt)} \varphi(t)dt \geq \int_0^{\min\{\frac{1}{3}\left(\begin{matrix} 1, \\ M(Ru,z_1,z,t)+M(Ru,z_1,z,t) \\ +[M(Ru,z_1,z,t)*1]\end{matrix}\right)\}} \varphi(t)dt$$

$$\int_0^{(Ru,_1,z,kt)} \varphi(t)dt \geq \int_0^{\min\{\left(\begin{matrix} 1, \\ MRu,_1,z,t\end{matrix}\right)\}} \varphi(t)dt$$

$$\int_0^{(Ru,_1,z,kt)} \varphi(t)dt \geq \int_0^{(Ru,_1,z,t)} \varphi(t)dt$$

Usinglemma2.9impliesthat$Ru = z_1 = Pu$

Since the pair(R,P)is weakly compatible ,so$Rz_1=RPu=PRu=Pz_1$

Again $\lim_{n\to\infty} Sy_n=z_1 \in Q(X)$ then $z_1=Qv$ for some $v\in Q$

Now we claim that $Qv=Sv$ then from (1),we have

$$\int_0^{M(Rx_n,Sv,z,kt)} \varphi(t)\,dt \geq \int_0^{\min\{\frac{1}{3}\left(\begin{array}{c} M(Px_n,Sv,z,t), \\ M(Ru,Pu,z,t)+M(Ru,Qy_n,z,t) \\ +[MRu,Qy_n,z,t*M(Qy_n,Pu,z,t)]\end{array}\right)\}} \varphi(t)\,dt$$

Taking limit $n\to\infty$,we have

$$\int_0^{(z_1,,z,kt)} \varphi(t)\,dt \geq \int_0^{\min\{\frac{1}{3}\left(\begin{array}{c} (z_1,,z,t), \\ M(z_1,z_1,z,t)+M(z_1,z_1,z,t) \\ +[Mz_1,z_1,z,t*M(z_1,z,z,t)]\end{array}\right)\}} \varphi(t)\,dt$$

$$\int_0^{(z_1,,z,kt)} \varphi(t)\,dt \geq \int_0^{\min\{\frac{1}{3}\left(\begin{array}{c} (z_1,,z,t), \\ M(z_1,z_1,z,t)+M(z_1,z_1,z,t) \\ +(z,z,1,1)\end{array}\right)\}} \varphi(t)\,dt$$

$$\int_0^{(z_1,,z,kt)} \varphi(t)\,dt \geq \int_0^{\min\{M(z_1,Sv,z,t),\,1\}} \varphi(t)\,dt$$

$$\int_0^{(z_1,,z,kt)} \varphi(t)\,dt \geq \int_0^{(z_1,v,z,t)} \varphi(t)\,dt$$

By using lemma2.9,we have

$$Qv=z_1=Sv$$

Since the pair(S,Q) is weakly compatible ,so$Qz_1=QSv=SQv=Sz_1$

Now we show that $Rz_1=z_1$ then from (1), we have

$$\int_0^{M(Rz_1,Sy_n,z,kt)} \varphi(t)\,dt \geq \int_0^{\min\{\frac{1}{3}\left(\begin{array}{c} M(Pz,Sy_n,z,t), \\ M(Rz,Pz,z,t)+M(Rz,Qy_n,z,t) \\ +[M(Rz,Qy_n,z,t)*M(Qy_n,Pz,z,t)]\end{array}\right)\}} \varphi(t)\,dt$$

Taking limit $n\to\infty$, we have

$$\int_0^{(Rz_{1,1},z,kt)} \varphi(t)\,dt \geq \int_0^{\min\left\{\begin{matrix}(z_{1,1},z,t),\\ \frac{1}{3}\left(M(Rz,z_1,z,t)+M(Rz_1,z_1,z,t)\right) \\ +[MRz_1,z_1,z,t*M(z_1,z_1,z,t)]\end{matrix}\right\}} \varphi(t)\,dt$$

$$\int_0^{(Rz_{1,1},z,kt)} \varphi(t)\,dt \geq \int_0^{\min\left\{\begin{matrix}1,\\ (MRz_{,1},z,t)\end{matrix}\right\}} \varphi(t)\,dt$$

$$\int_0^{(Rz_{1,1},z,kt)} \varphi(t)\,dt \geq \int_0^{(Rz_{1,1},z,t)} \varphi(t)\,dt$$

By using lemma2.9, we have

$$Rz_1 = z_1$$

Now we show that $Sz_1 = z_1$ then from (1), we have

$$\int_0^{M(Rx_n,Sz_1,z,kt)} \varphi(t)\,dt \geq \int_0^{\min\left\{\begin{matrix}(Px_{n,1},z,t),\\ \frac{1}{3}\left(M(Rx_n,Px_n,z,t)+M(Rx_n,Qz,z,t)\right) \\ +[MRx_n,Qz,z,t*M(Qz_1,Px_n,z,t)]\end{matrix}\right\}} \varphi(t)\,dt$$

Taking limit $n \to \infty$, we have

$$\int_0^{(z_{1,1},z,kt)} \varphi(t)\,dt \geq \int_0^{\min\left\{\begin{matrix}(z_{1,1},z,t),\\ \frac{1}{3}\left(M(z_1,z_1,z,t)+M(z_1,z_1,z,t)\right) \\ +[M(z_1,z_1,z,t)*M(z_1,z_1,z,t)]\end{matrix}\right\}} \varphi(t)\,dt$$

$$\int_0^{(z_{1,1},z,kt)} \varphi(t)\,dt \geq \int_0^{\min\left\{\begin{matrix}M(z_1,Sz_1,z,t),\\ 1\end{matrix}\right\}} \varphi(t)\,dt$$

$$\int_0^{(z_{1,1},z,kt)} \varphi(t)\,dt \geq \int_0^{(z_{1,1},z,t)} \varphi(t)\,dt$$

$$Sz_1 = z_1$$

$$Rz_1 = Pz_1 = Sz_1 = Qz_1 = z_1$$

Thus z_1 is common fixed point of P, Q, R and S.

Hence the uniqueness of the fixed point holds from equation (1).

Corollary3.2: Let P,Q,R and S be self –mapsofafuzzy2-

metricspaces(X,M,∗)satisfyingthefollowingcondition:

For some $F \in \Phi$ and $x, y, z \in X$, and for all t > 0 there exists a number k ∈ (0, 1)

such that

$$\int_0^{M(Rx,Sy,z,kt)} \varphi(t)dt \geq \int_0^{F\left\{\frac{1}{3}\left(\begin{matrix} M(Px,Sy,z,t), \\ M(Rx,Px,z,t)+M(Rx,Qy,z,t) \\ +[M(Rx,Qy,z,t)*M(Qy,Px,z,t)] \end{matrix}\right)\right\}} \varphi(t)dt$$

(1) Pairs(R, P) and (S, Q) satisfy common E. A. like property

(2) Pairs(R, P)and (S, Q) are weakly compatible.

Then P, Q,R and Shave a unique common fixed point in X.

Proof: The proof follows from The orem 3.1.

REFERENCES:

[1] AamirM.andMoutawakilD.El,Some new common fixed point the orem sun derstrict contractive conditions, J.Math Anal. Appl., 270 (2002), 181-188

[2] Cho S. H., International Mathematical forum 1, no. 10, p. 477-479.

(2006).[3]GahlerS., Math. Nachr.26 p. 115-148 (1983).

[4]GahlerS., Math. Nachr. 28 p. 1-43(1964).

[5]GahlerS.,Math. Nachr. 42 p.335-347 (1969).

[6] GeorgeA.andVeeramaniP.,"Onsomeresultsofanalysisforfuzzymetricspace"fuzzysetsand system, 64, p. 395-399, (1994).

[7] IsekiK.,SharmaP.L., SharmaB.K., Math.Japonica21,p. 67-70(1976).

[8] Kramosil,O.andMichalek,J.,Fuzzymetricstatisticalmetricspaces,Kybernetica(Praha),11,p. 336 – 344(1975).

[9] RhodesB.E."Twofixedpointtheoremformappingsatisfyingageneralcontractiveconditionof integral type".Int. J.Math. Sci.,3: p.4007-4013.

[10] SharmaS.,Southeast AsianBull.OfMath.26,p. 133-145(2002).

[11] WadhwaK., DubeyH. andJain R.,Impact of E.A.likepropertyon common fixedpointtheoremsin fuzzymetric spaces, J. of Adv. Stud. inTopol., 3, 1,2012, 52-59.

[12] Yadav D. S. and Thakur S. S., Common Fixed Points for R-Weakly

CommutingMappingsin Fuzzy2-MetricSpaces,Int. J.Contemp. Math. Scie.,8, 13,2013,609 –614.

[13] ZadehL.A.,FuzzySet,InformationandControl,8,p.338-353 (196